Detecting spurious jumps in high frequency data

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Preliminary version

Abstract

We study a technique to avoid spurious detection of jumps in high frequency data via an explicit thresholding on available test statistics. We show that it eliminates asymptotically all spurious jumps. Monte Carlo results also show that it performs well in finite samples. Our empirical investigation in the U.S. equity market reveals that the number of detected jumps amounts to around 30 a year, a reduction of more than 50%, and that we cannot reject the null hypothesis that the jump arrival times for two out of four stocks are driven by a simple Poisson process. Finally, we relate detected jumps to news releases.

JEL Classification: C10, C22, G10.

Keywords: Jumps, High frequency, Spurious detection, Jumps dynamics.

1 Introduction

Numerous methods to test for the presence of jumps in high frequency data have been introduced recently. These techniques are usually applied to test the null hypothesis of no jumps in a particular day, over a series of days. When implementing such a procedure, we are actually conducting a multiple (hypothesis) test, since we simultaneously test for the presence of jumps over several days (instead of just one). To illustrate the problem that arises, imagine we run a jump detection test at the 5% significance level, on hundred days where no single jump occurs. Then, on average, we erroneously select five days as containing a jump.

The first contribution of this paper is to propose a technique to avoid such spurious detection of jumps via an explicit thresholding on available test statistics. We prove that if we consider test statistics above a certain threshold level only, the likelihood of detecting spurious jumps disappears asymptotically. Monte Carlo results show that our approach behaves also well in finite samples. We perform an extensive simulation study, where we apply our thresholding technique to several jump detection statistics[1]. Spurious jumps are eliminated completely, and the power to detect genuine jumps is almost unaffected, for typical jump sizes. To illustrate the importance of eliminating spurious detections of jumps, we conduct an empirical study for the U.S. equity market. In particular, we collect high frequency returns from the Trades and Quotes (TAQ) database of stocks listed on the New York Stock Exchange (NYSE), over the three-year period of January 2006 to October 2008. We find that the typical average number of detected jumps shrinks from around 100 per year before thresholding (using the BNS ratio statistic at the one minute sampling frequency) to around 30.

The second contribution is to investigate the dynamic features of irregular jump arrivals and their associated market information. We believe our study is the first to investigate the dynamics of jumps quantitatively, by testing the distribution of jumps arrival times. Three years of high frequency data provide enough observations to perform a rigorous analysis, and we cannot reject the

[1] In this preliminary version, we focus on the ratio statistic of Barndorff-Nielsen and Shephard (2006). In the final version, we will provide results for other existing jump tests. Recently, formal tests have been developed for example in Aït-Sahalia and Jacod (forthcoming), Fan and Fan (2009), Lee and Mykland (2008), Jiang and Oomen (2008), Fan and Wang (2007), Carr and Wu (2003), Mancini (2003), or Johannes et al. (2004a, 2004b).

null hypothesis that jumps arrival times for two out of four stocks are driven by a simple Poisson process.

The remaining of the article is organized as follows. Section 2 presents our methodology to eliminate spurious detections of jumps, and shows results of our Monte Carlo study and of our empirical study. Section 3 investigates the dynamics of jumps arrival times.

2 Detecting spurious jumps

2.1 Available jump detection tests

Barndorff-Nielsen and Shephard (2006) introduce a test statistic based on the bipower variation of the asset price, which is asymptotically normal with mean zero under the null hypothesis of no jumps and converges in probability to some negative number depending on the jump sizes under the alternative hypothesis. Aït-Sahalia and Jacod (forthcoming) propose a test statistic, which converges to two different deterministic numbers that are independent of the dynamics of the diffusion process, depending on whether the sample path is present or absent of jumps. Fan and Fan (2009) develop a new test based on the method of Aït-Sahalia and Jacod (forthcoming). Their test statistic enjoys the same asymptotic properties, but has smaller variance. Lee and Mykland (2008) introduce and study a nonparametric test to detect jump arrival times up to the intra-day level. Their test statistic not only detects the presence of jumps but also gives estimates of the realized jump sizes in asset prices. Jiang and Oomen (2008) propose a test statistic which measures the impact of jumps on the third and higher order return moments and is directly related to the profit/loss function of a variance swap replication strategy. They also develop a modified version that retains power when applied to noisy data. Fan and Wang (2007) develop wavelet methods to estimate jump locations and jump sizes from a jump-diffusion process that is discretely observed with market microstructure noise. Other tests include Carr and Wu (2003), Mancini (2003), and Johannes et al. (2004a, 2004b).

2.2 Thresholding technique

Existing jump detection techniques are usually applied to test the presence of jumps on each day in a given period. As the tests are performed for many days simultaneously, we are actually conducting a multiple test, which by nature leads to making a proportion of spurious detections equal to the significance level of the individual tests. If for instance, as we do in the following section, we aim at studying the distribution of jumps arrivals over many years, these spurious detections can have a serious impact and lead to wrong conclusions.

One major contribution of our paper is a technique that allows to detect such spurious jumps, based on the following theoretical result developed in detail in the appendix. Denote by N the number of days in the study, and by n the number of observations per day used to compute each individual test statistic. We obtain a series of daily statistics which can be written as $(S_n^1, \ldots, S_n^N)$. For most available tests, under the null hypothesis of no jumps, the statistics converge to independent standard normal random variables. Theorem 1 of the appendix states that, under some technical conditions about the relative rate of convergence of n with respect to N and about the underlying price process, we get, under the null hypothesis of no jumps,

$$P\left[\sup_j |S_n^j| \leq \sqrt{2\log N}\right] \to 1, \quad \text{as} \quad N, n \to \infty.$$

This means that, if there are no jumps, the event that the largest and the smallest of the entries of the vector $(S_n^1, \ldots, S_n^N)$ stay within $\left[-\sqrt{2\log N}, \sqrt{2\log N}\right]$ becomes certain for large n and N.

Hence, if we neglect test statistics below the threshold $\sqrt{2\log N}$, then spurious detection of jumps becomes negligible with high probability[2]. The precise statement and the proof of the theorem are in the appendix, for a general test statistic, as well as for the specific examples of the statistics of Barndorff-Nielsen and Shephard (2006) and of Aït-Sahalia and Jacod (forthcoming).

2.3 Monte Carlo study

In this subsection, we examine the effectiveness of our approach using a Monte Carlo simulation. We detect jumps with the ratio statistic of Barndorff-Nielsen and Shephard (2006), and apply our thresholding technique to eliminate spurious detections in a second step[3]. The individual tests are performed at the 5% significance level. To generate data, we use the same model as in Aït-Sahalia

[2] $\sqrt{2\log N}$ is called the universal threshold.

[3] Preliminary version.

and Jacod (forthcoming) and Fan and Fan (2009):

$$dX_t/X_t = \sigma_t dW_t + J_t dN_t,$$
$$v_t = \sigma_t^2, \quad dv_t = \kappa(\beta - v_t)dt + \gamma v_t^{1/2} dB_t,$$

where W_t and B_t are both Brownian motions and $E[dW_t dB_t] = \rho dt$. The parameters are the same as in Aït-Sahalia and Jacod (forthcoming)—calibrated to be realistic for a liquid stock trading on the NYSE. $\beta = 0.4^2$, $\gamma = 0.5$, $\kappa = 5$ and $\rho = -0.5$. N_t is a Poisson process with intensity $\lambda = 20$ (jumps per year) chosen to correspond to what we observe in our data. The jump size is $J_t = J_s \beta^{1/2}$, that is J_s times the average value of the volatility level. We avoid the trivial situation where the jump sizes are very large and perform our study for $J_s = 0.1$, $J_s = 0.05$ (as in Fan and Fan (2009)), and $J_s = 0.01$. We approximate the diffusion process through the Euler scheme with an Euler tick of one second (although the prices are only recorded at lower frequencies). The time unit is one year. We discard the burn-in period, i.e., the first 500 data points of the whole series, to avoid the starting value effect. Each day consists of 6.5 hours of trading, that is 23,400 seconds. We simulate 1000 sample paths of prices over a three year period, corresponding to the time frame of our empirical study.

Table 1 displays the size of our new approach for combinations of different jump sizes, and different sampling frequencies ranging from 15 minutes to 30 seconds[4]. 'BNS' and 'BNS Thresh' indicate respectively results for the ratio statistic of Barndorff-Nielsen and Shephard (2006) without thresholding, and in the case when our thresholding technique is applied. Table 2 displays the corresponding power. Table 1 shows that our approach eliminates almost entirely the spurious detections. Even at the 15 minutes frequency, where the size is almost 10 percent before thresholding, we are able to detect almost every spurious jumps. Concerning the power, Table 2 shows that, even for medium-sized jumps and before thresholding, the ability to detect jumps deteriorates significantly for 5 minutes and lower sampling frequencies. However, if we sample every one minute or every 30 seconds, the power remains almost unaffected by our thresholding technique, e.g., with medium-size jumps and one minute sampling frequency, the power decreases only by 5.6 percentage points to 92.3%. In the remaining of the study we use the one minute sampling frequency .

[4]The size is the test's probability of incorrectly rejecting the null hypothesis, i.e., the false positive rate.

[Tables 1 and 2]

2.4 Empirical results

[Figure ??]

In the current preliminary version, we conduct our analysis on four stocks traded on the NYSE, i.e., Citigroup (C), General Electric (GE), IBM (IBM), and Walmart (WMT)[5]. The data is extracted from the TAQ database and consists of all trades that took place on the primary exchange (NYSE) over the period January 2006 through October 2008. We also retain all trades executed through NYSE Direct+ (indicated by sale condition 'E'). Towards the end of the sample period, these latter trades constitute an important proportion of the trading volume. We consider only trades with a time stamp between 9:40 am and 3:55 pm. The original data are pre-processed eliminating obvious data errors, such as transaction prices reported at zero, transaction times that are out of order, trades with a non-zero correction indicator, trades with a non-empty sale condition (different from 'E'), "bounce back" outliers.

[Table 3]

Figure **??** illustrates the thresholding process for the IBM stock during the first half of the year 2007, with a sampling frequency of one minute. For each day in the sample, the points show the value of the ratio statistic of Barndorff-Nielsen and Shephard (2006). The dash-dotted line displays the cutoff level below which test statistics are set to zero by our thresholding technique, and the dotted line corresponds to the critical value. The final results are shown by the asterisks, and the spurious jumps are depicted by a circle. We see that an important proportion of jumps is eliminated by our thresholding technique. Table 3 shows that the average number of jumps per year identified by the BNS statistic drops from above 100, to less than 50 when we apply our thresholding technique. Figure **??** provides examples of spurious jumps detected by our methodology (panels (a) and (b)). We see that the transaction prices show no discontinuity. Figure **??** also displays examples of days with genuine jumps (panels (c) and (d)).

[Figure **??**]

[5] The final version will be based on the thirty DJIA stocks.

3 Jump distribution and relation to news releases

3.1 Distribution of jump occurrences

[Tables 4, 5, and 6]

This section investigates the dynamics of jump occurrences. We study the distribution of arrival times between successive jumps. Specifically, we use the Kolmogorov-Smirnov test to test whether arrival times follow an exponential, a Weibull, or a lognormal distribution[6]. Table 4 displays the p-values of the null hypothesis that arrival times follow an exponential duration for four stocks traded on the NYSE[7]. For Citigroup and IBM, when our thresholding technique is applied, the null that jumps occurrences are driven by an exponential distribution is not rejected. Figures **??** and **??** show the histograms of the durations between two consecutive jumps, for respectively IBM and General Electric sampled at the one minute frequency. Results for the BNS statistic and for our thresholding methodology are displayed in respectively panel (a) and panel (b). The figures also plot the densities of fitted exponential, lognormal, and Weibull densities. We see that the exponential distribution is rejected because of the gap at very short durations between jumps.

For some stocks, we cannot reject the null hypothesis that jump arrival times are driven by a simple Poisson process.

[Figures **??** and **??**]

3.2 Factiva

In this section we investigate the relation of jumps to news releases. We use Factiva, a financial news database that includes the Reuters and Dow Jones newswires, and news from the *Financial Times* and the *Wall Street Journal*.

[6] As we estimate the parameters of the distribution tested against the data, using the Kolmogorov-Smirnov test directly is not accurate. Therefore, we use critical values obtained by bootstrap. In the appendix we also perform a Monte Carlo study that illustrates the good statistical properties of the Kolmogorov-Smirnov procedure in a setting similar to our empirical application.

[7] The final version will be based on the thirty DJIA stocks.

Appendices

A Proof of theorem

Under the null hypothesis of no jumps the asymptotic distribution of jump test statistics can be shown to converge to independent standard normal random variables. These results follows from showing asymptotic negligibility of the drift contributions and application of a CLT for triangular arrays of martingale differences.

For each integer $n \geq 1$, let the real-valued random variables $X_{n1}^j, \dots, X_{nn}^j$, $1 \leq j \leq N$, form N square integrable martingale difference sequences w.r.t. the σ-fields $\mathcal{F}_{n0}^j \subset \mathcal{F}_{n1}^j \subset \dots \subset \mathcal{F}_{nn}^j$, that is, suppose that X_{ni}^j is measurable w.r.t. $\mathcal{F}_{ni}^j$ with $E[(X_{n1}^j)^2] < \infty$ and $E[X_{ni}^j | \mathcal{F}_{ni}^j] = 0$ a.s. for all n, i and j. The CLT is applied to quantities which can be written as $S_n^j = \sum_{i=1}^{n} X_{ni}^j$. In the following theorem we show that the event that the largest and the smallest of the entries of the vector $(S_n^1, \dots, S_n^N)$ stay within $[-\sqrt{2\log N}, \sqrt{2\log N}]$ becomes certain for large n and N. We use two conditions on higher moments, which imply the conditions to apply the CLT for triangular arrays of martingale differences when n goes to infinity, and require that N is not too large w.r.t. the asymptotics in n.

Theorem 1. *Let* $S_n^j = \sum_{i=1}^{n} X_{ni}^j$, $1 \leq j \leq N$. *If, for* $0 < \gamma < \infty$,

$$L_{n,2\gamma}^j = E\left[\sum_{i=1}^{n} |X_{ni}^j|^{2+2\gamma}\right] \to 0, \quad as \quad n \to \infty, \tag{1}$$

$$M_{n,2\gamma}^j = E\left[|\sum_{i=1}^{n} E\left[(X_{ni}^j)^2 | \mathcal{F}_{ni}^j\right] - 1|^{1+\gamma}\right] \to 0, \quad as \quad n \to \infty, \tag{2}$$

and

$$(1 + \sqrt{2\log N})^{3+6\gamma} N \leq \alpha (L_{n,2\gamma}^j + M_{n,2\gamma}^j)^{-1}, \tag{3}$$

with $\alpha > 0$. *Then,*

$$P\left[\sup_j |S_n^j| \leq \sqrt{2\log N}\right] \to 1, \quad as \quad N, n \to \infty. \tag{4}$$

Proof. Conditions (1) and (2) imply the conditions of the CLT for triangular arrays of martingale differences, and we get the weak convergence of the distribution $P\left[S_n^j \leq x\right]$ to the standard normal distribution $\Phi(x)$ as $n \to \infty$.

Now $P\left[\sup_j |S_n^j| \le \sqrt{2\log N}\right] = P\left[|S_n^1| \le \sqrt{2\log N}, \ldots, |S_n^N| \le \sqrt{2\log N}\right] =$
$\prod_{j=1}^{N} P\left[|S_n^j| \le \sqrt{2\log N}\right]$ by independence.

From Grama (1997) Theorem 2.1, Condition (3) ensures that we can use exact bounds for the departure from normality of $P\left[S_n^j \ge \sqrt{2\log N}\right]$ and $P\left[S_n^j \le -\sqrt{2\log N}\right]$ (see also Hauesler (1988) Theorem 2 for exact uniform bounds, and Lipster and Shiryaev (1989) Section 5.7 Theorems 1 and 2 for uniform bounds, i.e., Berry-Esseen type bounds, instead of the exact nonuniform bounds for moderate deviations that we use here), so that

$$\begin{aligned}\prod_{j=1}^{N} P\left[|S_n^j| \le \sqrt{2\log N}\right] &= \prod_{j=1}^{N}\left(1 - P\left[S_n^j \ge \sqrt{2\log N}\right] - P\left[S_n^j \le -\sqrt{2\log N}\right]\right)\\ &= \prod_{j=1}^{N}\left[1 - 2\Phi(-\sqrt{2\log N})\left\{1 + R^j(\alpha,\gamma,N)\right\}\right],\end{aligned}$$

where the remainder term is

$$R^j(\alpha,\gamma,N) = \theta C(\alpha,\gamma)\left\{(1+\sqrt{2\log N})^{3+6\gamma} N(L^j_{n,2\gamma} + M^j_{n,2\gamma})\right\}^{1/(3+2\gamma)}$$

with $|\theta| < 1$ and $C(\alpha,\gamma)$ being a constant only depending on α and γ. Using $\Phi(-\sqrt{2\log N}) \le \phi(\sqrt{2\log N})/\sqrt{2\log N}$ with ϕ denoting the density of the standard normal distribution, we deduce the stated result from

$$\prod_{j=1}^{N}\left[1 - 2\Phi(-\sqrt{2\log N})\right] = \left[1 - \frac{2}{\sqrt{2\pi}\sqrt{2\log N}N}\right]^N \to 1, \quad \text{as} \quad N \to \infty,$$

and the asymptotic negligibility of the contribution of the remainder term as $N, n \to \infty$ since $R^j(\alpha,\gamma,N)$ is bounded by $\theta C(\alpha,\gamma)\alpha^{3+6\gamma}$ because of (3). $\square$

Remarks: Condition (3) is rather weak as clearly illustrated in the case of independent random variables by Grama (1997). Let $X^j_{ni} = \eta^j_i/\sqrt{n}$, where η^j_i form N given independent sequences of i.i.d. random variables which satisfy $E[\eta^j_1] = 0$, $E[(\eta^j_1)^2] = 1$, $m_{2\gamma} = E[|\eta^j_1|^{2+2\gamma}] < \infty$ with $0 < \gamma < \infty$. In this case $M^j_{n,2\gamma} = 0$ and $L^j_{n,2\gamma} = n^{-\gamma} m_{2\gamma}$. This means that in order to satisfy (3) we can afford N much lower than n since α and γ can be taken sufficiently large in practice.

Ait-Sahalia-Jacod test: We can write the test statistic of Ait-Sahalia and Jacod (2007) in the above form using $X_{ni} := \dfrac{(\hat{V}^c_n)^{-1/2}\left(|\Delta^n_{ki}X|^p - |\Delta^n_i X|^p k^{p/2-1}\right)}{\hat{B}(p,\delta_n)}$.

Since

$$|X_{ni}|^{2+2\gamma} \leq \frac{(\hat{V}_n^c)^{-(1+\gamma)} \sum_{l=0}^{\infty} \begin{pmatrix} 2+2\gamma \\ l \end{pmatrix} |\Delta_{ki}^n X|^{p(2+2\gamma-l)} |\Delta_i^n X|^{pl} k^{(p/2-1)l}}{\hat{B}(p,\delta_n)^{2+2\gamma}},$$

where $\begin{pmatrix} 2+2\gamma \\ l \end{pmatrix} = \frac{1}{l!}\prod_{j=0}^{l-1}(2+2\gamma-j)$, Condition (1) holds from the convergence of $\Delta_n^{-1/2}\Delta_n^{1-p(1+\gamma)} \sum_{i=1}^{n} |\Delta_{ki}^n X|^{p(2+2\gamma-l)}|\Delta_i^n X|^{pl} k^{(p/2-1)l}$, $p \geq 2$, in law to Gaussian variables for X continuous, the equality $\Delta_n^{-(1+\gamma)} = \Delta_n^{-1/2}\Delta_n^{1-p(1+\gamma)}\Delta_n^{p+\gamma(p-1)-3/2}$, and $\Delta_n \to 0$. Condition (2) holds since $\dfrac{\sum_{i=1}^{n} E\left[\left(|\Delta_{ki}^n X|^p - |\Delta_i^n X|^p k^{(p/2-1)}\right)^2 |\mathcal{F}_{ni}\right]}{\hat{B}(p,\Delta_n)^2}$ converges to V_n^c (see Ait-Sahalia and Jacod (2007), proof of Theorem 4).

Barndorff-Nielsen-Sheppard test: We can write the test statistic of Barndorff-Nielsen and Sheppard (2006) in the above form using $X_{ni} := (\vartheta\mu_1^{-4}\Delta_n QPV)^{-1/2}\left(\mu_1^{-2}|\Delta_i^n X||\Delta_{i-1}^n X| - |\Delta\right.$
Since

$$|X_{ni}|^{2+2\gamma} \leq (\vartheta\mu_1^{-4}\Delta_n QPV)^{-1/2} \sum_{l=0}^{\infty} \begin{pmatrix} 2+2\gamma \\ l \end{pmatrix} \left(|\Delta_i^n X||\Delta_{i-1}^n X|\right)^{2(2+2\gamma-l)} |\Delta_i^n X|^{2l},$$

where $\begin{pmatrix} 2+2\gamma \\ l \end{pmatrix} = \frac{1}{l!}\prod_{j=0}^{l-1}(2+2\gamma-j)$, Condition (1) holds from the convergence of $\Delta_n^{-1/2}\Delta_n^{1-2(1+\gamma)} \sum_{i=2}^{n} \left(|\Delta_i^n X||\Delta_{i-1}^n X|\right)^{2(2+2\gamma-l)} |\Delta_i^n X|^{2l}$, $p \geq 2$, in law to Gaussian variables for X continuous, the equality $\Delta_n^{-(1+\gamma)} = \Delta_n^{-1/2}\Delta_n^{1-2(1+\gamma)}\Delta_n^{2+\gamma-3/2}$, and $\Delta_n \to 0$. Condition (2) holds since $\sum_{i=2}^{n} E\left[\left(\mu_1^{-2}|\Delta_i^n X||\Delta_{i-1}^n X| - |\Delta_i^n X|^2\right)^2 |\mathcal{F}_{ni}\right]$ converges to $A(4)_t$ (see Barndorf-Nielsen et al. (2006), proof of Proposition 4.2).

References: Grama (1997), Hauesler (1988), Lipster and Shiryayev (1989).

B Jumps dynamics distribution Monte Carlo study

In order to verify the statistical properties of the Kolmogorov-Smirnov test in a setting corresponding to our empirical application of Section 3.1, we simulate 1000 sample paths of prices over a three year period using the model of Section 2.3. In the first case, jumps are generated from an exponential distribution with parameter $\lambda = 20$ (jumps per year). In the second case, jumps are generated from an Weibull distribution with parameters (20,3). Table 7 shows that the power and the size of the test are excellent.

[Table 7]

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		Jumps size		
BNS		$0.1\beta^{1/2}$	$0.05\beta^{1/2}$	$0.01\beta^{1/2}$
30 sec	No thresholding	5.1	4.9	5.1
	FDR thresholding	1.6	1.6	0.0
	Universal threshold	0.0	0.0	0.0
5 min	No thresholding	5.5	5.4	5.3
	FDR thresholding	2.0	1.5	0.0
	Universal threshold	0.0	0.0	0.0
15 min	No thresholding	5.8	5.7	5.7
	FDR thresholding	2.0	0.5	0.0
	Universal threshold	0.0	0.0	0.0
ABD		$0.1\beta^{1/2}$	$0.05\beta^{1/2}$	$0.01\beta^{1/2}$
30 sec	No thresholding	5.4	5.2	5.4
	FDR thresholding	0.3	0.3	0.2
	Universal threshold	0.0	0.0	0.0
5 min	No thresholding	6.3	6.3	6.2
	FDR thresholding	0.7	0.6	0.1
	Universal threshold	0.1	0.1	0.1
15 min	No thresholding	8.0	7.6	7.6
	FDR thresholding	1.3	1.2	0.3
	Universal threshold	0.2	0.2	0.2
AJ		$0.1\beta^{1/2}$	$0.05\beta^{1/2}$	$0.01\beta^{1/2}$
30 sec	No thresholding	14.1	14.2	14.0
	FDR thresholding	7.8	7.8	5.0
	Universal threshold	0.5	0.5	0.4
5 min	No thresholding	10.6	11.2	10.7
	FDR thresholding	1.9	1.9	0.7
	Universal threshold	0.0	0.0	0.0
15 min	No thresholding	6.5	6.7	6.9
	FDR thresholding	0.0	0.0	0.0
	Universal threshold	0.0	0.0	0.0

Table 1: Monte Carlo: Size

		Jumps size		
BNS		$0.1\beta^{1/2}$	$0.05\beta^{1/2}$	$0.01\beta^{1/2}$
30 sec	No thresholding	99.9	99.9	16.4
	FDR thresholding	99.9	99.9	0.5
	Universal threshold	99.9	99.9	0.4
5 min	No thresholding	98.7	83.6	6.3
	FDR thresholding	98.5	71.6	0.0
	Universal threshold	97.0	38.7	0.0
15 min	No thresholding	89.7	40.1	6.2
	FDR thresholding	82.4	14.0	0.0
	Universal threshold	43.7	3.1	0.0
ABD		$0.1\beta^{1/2}$	$0.05\beta^{1/2}$	$0.01\beta^{1/2}$
30 sec	No thresholding	100.0	100.0	71.4
	FDR thresholding	100.0	100.0	48.0
	Universal threshold	100.0	100.0	30.2
5 min	No thresholding	99.9	99.7	8.0
	FDR thresholding	99.9	98.5	0.1
	Universal threshold	99.9	94.1	0.1
15 min	No thresholding	99.6	72.6	8.0
	FDR thresholding	98.1	44.4	0.2
	Universal threshold	92.9	22.7	0.1
AJ		$0.1\beta^{1/2}$	$0.05\beta^{1/2}$	$0.01\beta^{1/2}$
30 sec	No thresholding	99.9	99.8	35.9
	FDR thresholding	99.9	99.6	21.2
	Universal threshold	99.9	97.2	6.0
5 min	No thresholding	64.1	50.6	11.2
	FDR thresholding	15.5	17.5	0.6
	Universal threshold	0.7	0.4	0.0
15 min	No thresholding	12.1	11.9	6.4
	FDR thresholding	0.0	0.0	0.0
	Universal threshold	0.0	0.0	0.0

Table 2: Monte Carlo: Power

	BNS		ABD		AJ	
Thresholding:	None	FDR	None	FDR	None	FDR
MMM	41.0	9.7	105.0	54.3	10.7	0.0
AA	35.7	5.7	104.3	52.7	9.7	0.0
AXP	37.0	6.7	108.3	53.3	5.3	0.0
T	41.0	7.7	100.0	45.3	8.7	0.0
BAC	36.7	4.7	101.0	40.0	7.0	0.0
BA	37.7	10.3	116.3	57.3	7.0	0.0
CAT	38.0	6.7	114.0	58.3	2.7	0.0
CVX	33.3	3.7	82.7	33.0	7.7	0.0
KO	40.3	7.7	101.3	45.3	6.3	0.3
DD	51.3	14.7	111.3	58.0	6.0	0.0
XOM	31.0	1.3	73.0	32.7	11.0	0.0
GE	40.0	8.3	91.7	37.7	7.7	0.0
HPQ	46.7	11.7	111.7	57.3	6.3	0.0
HD	38.3	8.3	106.3	52.0	6.3	0.0
IBM	41.7	8.0	98.3	44.3	8.0	0.0
JNJ	35.0	11.0	109.3	56.7	12.7	0.0
JPM	39.0	6.3	110.0	51.3	6.3	0.0
KFT	61.3	27.3	138.7	85.3	7.0	0.0
MCD	41.3	8.3	106.7	48.7	10.3	0.0
MRK	46.3	11.7	120.0	57.3	8.0	0.0
PFE	45.3	9.0	92.3	40.7	9.7	0.0
PG	33.3	5.3	107.0	47.0	8.7	0.0
UTX	36.0	7.7	102.3	45.0	8.7	0.0
VZ	50.0	15.3	109.3	56.7	9.0	0.0
WMT	38.0	9.0	99.7	47.3	9.0	0.0
DIS	49.0	9.0	107.3	45.3	9.3	0.0
mean	40.9	9.0	104.9	50.1	8.0	0.0
median	39.5	8.3	106.5	50.0	8.0	0.0
min	31.0	1.3	73.0	32.7	2.7	0.0
max	61.3	27.3	138.7	85.3	12.7	0.3

Table 3: Average number of jumps per year, obtained with the BNS test and with our thresholding methodology. Five minutes sampling frequency. Tests performed over 2006–2008.

	BNS		ABD		AJ	
Thresholding:	None	FDR	None	FDR	None	FDR
MMM	0.00	0.13	0.00	0.00	0.70	-
AA	0.00	0.34	0.00	0.00	0.59	-
AXP	0.00	0.18	0.00	0.00	0.43	-
T	0.00	0.59	0.00	0.00	0.19	-
BAC	0.00	0.20	0.00	0.00	0.44	-
BA	0.00	0.98	0.00	0.00	0.01	-
CAT	0.00	0.80	0.00	0.00	0.62	-
CVX	0.01	0.16	0.00	0.01	0.91	-
KO	0.00	0.01	0.00	0.00	0.63	-
DD	0.00	0.63	0.00	0.00	0.02	-
XOM	0.00	0.36	0.00	0.02	0.53	-
GE	0.00	0.72	0.00	0.00	0.35	-
HPQ	0.00	0.15	0.00	0.00	0.39	-
HD	0.00	0.23	0.00	0.00	0.98	-
IBM	0.00	0.90	0.00	0.00	0.88	-
JNJ	0.01	0.44	0.00	0.00	0.55	-
JPM	0.00	0.71	0.00	0.00	0.40	-
KFT	0.00	0.14	0.00	0.00	0.21	-
MCD	0.00	0.23	0.00	0.00	0.64	-
MRK	0.00	0.61	0.00	0.00	0.45	-
PFE	0.00	0.14	0.00	0.00	0.59	-
PG	0.01	0.36	0.00	0.00	0.53	-
UTX	0.01	0.82	0.00	0.00	0.24	-
VZ	0.00	0.68	0.00	0.00	0.68	-
WMT	0.00	0.71	0.00	0.00	0.36	-
DIS	0.00	0.78	0.00	0.00	0.92	-
Prop. reject. (%)	100.0	3.8	100.0	100.0	7.7	-

Table 4: p-value of Kolmogorov-Smirnov test for the null hypothesis that jump arrival times follow an exponential distribution. Five minutes sampling frequency. Tests performed over 2006–2008.

	BNS		ABD		AJ	
Thresholding:	None	FDR	None	FDR	None	FDR
MMM	0.00	0.12	0.00	0.00	0.40	-
AA	0.02	0.30	0.00	0.00	0.11	-
AXP	0.01	0.70	0.00	0.00	0.31	-
T	0.00	0.24	0.00	0.00	0.41	-
BAC	0.06	0.72	0.00	0.01	0.31	-
BA	0.01	0.39	0.00	0.00	0.03	-
CAT	0.00	0.43	0.00	0.00	0.57	-
CVX	0.01	0.36	0.00	0.01	0.79	-
KO	0.00	0.83	0.00	0.00	0.37	-
DD	0.00	0.14	0.00	0.00	0.49	-
XOM	0.01	0.33	0.00	0.05	0.82	-
GE	0.00	0.74	0.00	0.01	0.53	-
HPQ	0.01	0.05	0.00	0.00	0.25	-
HD	0.01	0.99	0.00	0.00	0.96	-
IBM	0.00	0.27	0.00	0.00	0.48	-
JNJ	0.03	0.08	0.00	0.00	0.78	-
JPM	0.00	0.65	0.00	0.00	0.51	-
KFT	0.00	0.22	0.00	0.00	0.83	-
MCD	0.02	0.47	0.00	0.00	0.39	-
MRK	0.00	0.46	0.00	0.00	0.70	-
PFE	0.00	0.04	0.00	0.02	0.51	-
PG	0.01	0.35	0.00	0.00	0.67	-
UTX	0.00	0.40	0.00	0.00	0.14	-
VZ	0.00	0.53	0.00	0.00	0.79	-
WMT	0.05	0.17	0.00	0.00	0.21	-
DIS	0.00	0.96	0.00	0.00	0.90	-
Prop. reject. (%)	92.3	3.8	100.00	96.2	3.8	-

Table 5: p-value of Kolmogorov-Smirnov test for the null hypothesis that jump arrival times follow a lognormal distribution. Five minutes sampling frequency. Tests performed over 2006–2008.

	BNS		ABD		AJ	
Thresholding:	None	FDR	None	FDR	None	FDR
MMM	0.00	0.00	0.00	0.00	0.60	-
AA	0.01	0.31	0.00	0.00	0.40	-
AXP	0.01	0.36	0.00	0.00	0.65	-
T	0.00	0.44	0.00	0.00	0.26	-
BAC	0.03	0.22	0.00	0.00	0.13	-
BA	0.00	0.83	0.00	0.00	0.01	-
CAT	0.01	0.85	0.00	0.00	0.42	-
CVX	0.03	0.18	0.00	0.02	0.84	-
KO	0.00	0.18	0.00	0.00	0.42	-
DD	0.00	0.73	0.00	0.00	0.08	-
XOM	0.00	0.64	0.00	0.01	0.70	-
GE	0.00	0.74	0.00	0.00	0.30	-
HPQ	0.00	0.41	0.00	0.00	0.37	-
HD	0.00	0.33	0.00	0.00	0.91	-
IBM	0.00	0.90	0.00	0.00	0.77	-
JNJ	0.03	0.24	0.00	0.00	0.25	-
JPM	0.00	0.50	0.00	0.00	0.36	-
KFT	0.00	0.16	0.00	0.00	0.18	-
MCD	0.00	0.37	0.00	0.00	0.16	-
MRK	0.00	0.30	0.00	0.00	0.21	-
PFE	0.00	0.24	0.00	0.00	0.98	-
PG	0.00	0.34	0.00	0.00	0.56	-
UTX	0.00	0.65	0.00	0.00	0.09	-
VZ	0.00	0.35	0.00	0.00	0.38	-
WMT	0.02	0.79	0.00	0.00	0.43	-
DIS	0.00	0.34	0.00	0.00	0.75	-
Prop. reject. (%)	100.0	3.8	100.00	100.0	3.8	-

Table 6: p-value of Kolmogorov-Smirnov test for the null hypothesis that jump arrival times follow a Weibull distribution. Five minutes sampling frequency. Tests performed over 2006–2008.

Frequency		Duration between jumps Exponenetial	Weibull
30 sec	BNS Thresh	4	100
	BNS	5	100
1 min	BNS Thresh	6	100
	BNS	6	100
5 min	BNS Thresh	42	92
	BNS	6	84
true jumps		5	100

Table 7: Proportion of rejections of the null hypothesis that arrival times follow an exponential distribution (using the KS test), when arrival times are generated from respectively an exponential ($\lambda = 20$), and a Weibull (with parameters (20,3)) distribution (in %).